Section 2-1, Mathematics 108

Functions

Some Definitions:

1) In it's most general sense the term **function** a mapping of elements between two sets.

Given the set $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$

We use the script letter *f* to mean the function that takes elements from *A* to *B*

$$x \to f(x)$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

Note that each element *x* of *A* is mapped to exactly one element of *B*.

It is quite permissible for more than one element of *A* to be mapped to the same element of *B*.

$$x \to f(x)$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$$

The following is not a function, why?

$$\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \begin{matrix} \to \\ \to \\ \to \end{matrix} \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$$

2) The set A is known as the **Domain** of the function.

3) The symbol **$f(x)$** is called the **value of f at x**, or the **image of x**.

4) The set of all the $f(x)$'s , $\{f(x) \mid x \in A\}$ is called the **range of f**.

5) We call a variable such as x that represents an element of A the **independent variable.**

6) We call a variable that represents $f(x)$ the **dependent variable.**

So if we write

$$y = f(x)$$

x is the independent variable and y is the dependent variable.

Some Notes

1) The sets A and B can be the same set.

2) Two functions could have the same mapping but be different functions.

Example:

The **identity** mapping is a mapping from a set A onto itself.
It is the mapping where for all $x \in A$, $f(x) = x$.

Note that the identity mapping on the integers and the identity mapping on the real numbers are two different functions.

3) We most often use the notation $f(x)$ to mean a function, but when we are dealing with more than one function, we may use $g(x)$, $h(x)$ or any other convenient letter or letters.

Ways to describe a function

1) Verbally

Example:
"The function which maps each student-id at USF to that particular Student".

In this case A is as set of student-id numbers and B is the set of students at USF.

2) Using a specific listing

x	$f(x)$
1	3
2	9
3	27

2) Using a table

Cost to send a 1st class letter in the US

ounces	price
≤ 1 ounce	.49
> 1 and ≤ 2 ounces	.70
> 2 and ≤ 3 ounces	.91

3) Using an algebraic expression

An example is a function that returns the area of a circle given it's radius.

$$A(r) = \pi r^2$$

Note that without explicitly knowing the domain of this function, we can assume it consists of all valid real numbers greater than 0.

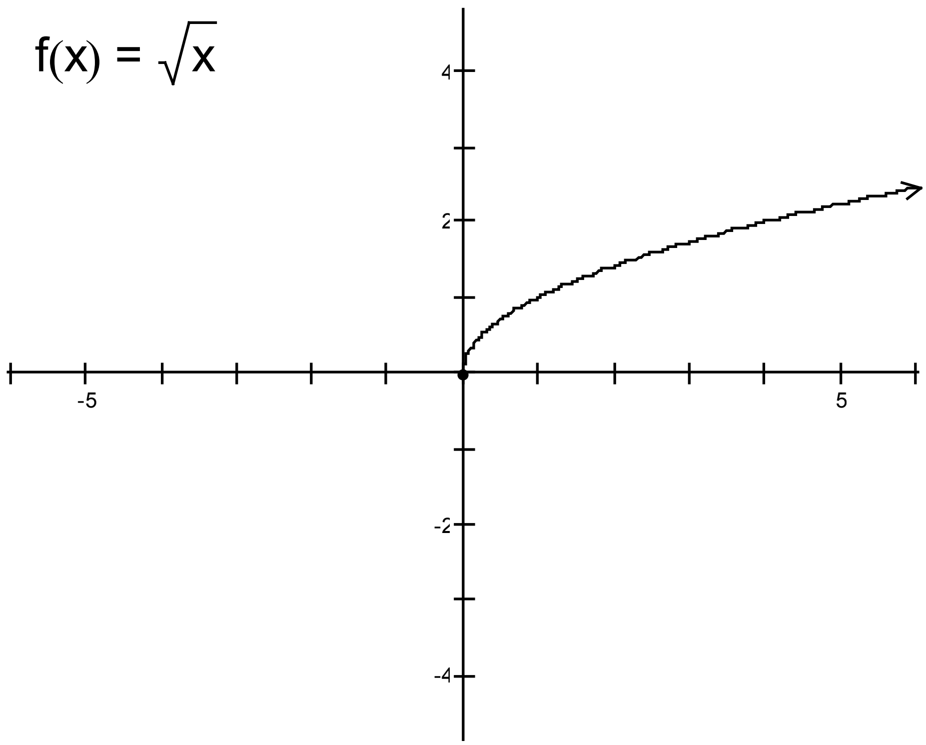
This will be one of the most commonly used ways to describe a function

4) Using a Piecewise definition

Here is an example:

$$C(x)=\begin{cases} 39 & 0\le x\le 2 \\ 39+15(x-2) & x>2 \end{cases}$$

5) Using a graph



Note that it would not be possible to describe this function completely using a listing.

Evaluating a function described algebraically

If we have a function

$$f(x) = 3x^2 + x - 5$$

that we want to evaluate at x=2, we just plug in the value to the expression.
We can write this as follows:

$$f(2) = 3(2)^2 + 2 - 5 = 9$$

so

$$f(2) = 9$$

More on the domain of a function

Like the domain of an algebraic expression, the domain of a function may be stated explicitly, eg.

$$f(x) = x^2 \quad 0 \le x \le 5$$

If the domain is not otherwise stated, we assume it is a maximal subset of the reals. That is the domain is the reals minus any values that are undefined.

Example:

a) $f(x) = \dfrac{1}{x(x-1)}$ What is the domain?

b) $g(x) = \sqrt{9 - x^2}$ What is the domain?

c) $h(t) = \dfrac{t}{\sqrt{t+1}}$ What is the domain?